A LECTURE ON INTEGRATION BY PARTS

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Abstract
Integration by parts (IP) has developed a bad reputation. While it allows us to manage a wide variety of integrals when other methods fall short, its implementation can be thought of as plodding and confusing. However, readers familiar with the tabular method for IP know that it can significantly streamline computations and promote creativity. In this paper the flexibility of the tabular method is explored by approaching it from a different direction and examining a number of examples. This is done in order to showcase the notion that the tabular method can be applied whenever IP is used. The key idea is that each new row represents a new integral and, hence, tables are constructed one row at a time.

Keywords: Calculus; integration by parts; tabular method; DI method; tic-tac-toe method; Taylor’s formula

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1. Building the table

The tabular method for integration by parts (IP) has been documented since at least the 1940s (see [2]) and various versions of it can often be found in modern calculus textbooks (see, for instance, [1, Section 7.2]). Moreover, it has been used to compute certain types of integrals, as seen in [2], [4], [5], [8], and [10].

The tabular approach to IP, as developed here, allows us to readily compute all sorts of integrals when an application of IP is appropriate. Additionally, this tabular method lets us quickly learn from poor choices and neatly derive formulae corresponding to important results, such as Taylor’s formula with integral remainder (see Theorem 1 below and [4]). The approach taken here emphasizes the idea that each row represents an integral stemming from an application of IP and, therefore, proceeding one row at a time yields the best result.

The formula for IP (sometimes called ‘ultra-violet voodoo’) has a straightforward justification. Recall the product rule for derivatives: for suitable functions u and v of a real variable x, we have

\[(uv)' = uv' + u'v.\]

By integrating both sides of this equation, applying the definition of an antiderivative, and changing variables accordingly, we have

\[uv = \int u \, dv + \int v \, du.\]

A slight rearrangement produces the formula (‘ultra-violet voodoo’)

\[\int u \, dv = uv - \int v \, du.\]


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In general, the idea behind IP is to let \( \int u \, dv \) denote a given integral in the hopes that a new integral \( \int v \, du \) can be readily determined or managed. The pair \( u \) and \( dv \) are chosen so that their product represents the given integrand; and the pair \( v \) and \( du \) are determined by \( \int dv = \int v_0 \, dx \) (usually excluding a constant) and \( u' = du/\, dx \), respectively.\(^1\) It is important to note that, like substitution, IP always leaves an integral to be resolved.

In order to motivate the use of a tabular approach and to build the table, suppose that we want to iterate the IP as follows (note that this will not always be the case). Given \( \int u \, dv \), let \( u_1 := u \), \( v_0 \, dx := dv \), and \( v_1 := \int \, dv = \int v_0 \, dx \). For each integer \( j \geq 2 \), let \( u_j := u'_{j-1} \) and \( v_j := \int v_{j-1} \, dx \). (We assume throughout that functions behave well enough.) Then, for each integer \( n \geq 2 \), we have

\[
\int u \, dv = uv - \int v \, du \\
= u_1 v_1 - u_2 v_2 + \int v_2 \, du_2 \\
= \cdots \\
= u_1 v_1 - u_2 v_2 + \cdots + (-1)^{n-1} u_n v_n + (-1)^n \int v_n \, du_n. 
\]

(1)

For a full justification of (1), see [2] and [6]. Note that in (1) we suppress the addition of a constant. Throughout the paper, \( C \) and \( C_0 \) denote constants.

If we were to iterate the IP as in (1), when would we stop? The answer depends on how we want to handle the integrals \( (-1)^j \int v_j \, du_j \), as each one is generated. The simplest case occurs when some \( (-1)^j \int v_j \, du_j \) are readily computed, but the more difficult cases prove to be the most interesting. (See Examples 4 and 5 below.)

To create a table that leads to (1), label three columns \(+/-\), \( u \), and \( dv \). We begin with a ‘+’ sign in the first row of the \(+/-\) column, a chosen \( u_1 \) in the \( u \) column, and \( v_0 \) (where \( dv = v_0 \, dx \) and the differential \( dx \) is suppressed) in the \( dv \) column. With each new row, alternate (alt.) between ‘+’ and ‘-’ in the \(+/-\) column, differentiate (diff.) the function in the \( u \) column, and integrate (int.) the function in the \( dv \) column. With each row, decide how to proceed based on the integral of the product of the terms in that row.

<table>
<thead>
<tr>
<th>(+/-) (alt.)</th>
<th>( u ) (diff.)</th>
<th>( dv ) (int.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>( u_1 )</td>
<td>( v_0 )</td>
</tr>
<tr>
<td>-</td>
<td>( u_2 )</td>
<td>( v_1 )</td>
</tr>
<tr>
<td>+</td>
<td>( u_3 )</td>
<td>( v_2 )</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>(-1)^{n-1}</td>
<td>( u_n )</td>
<td>( v_{n-1} )</td>
</tr>
<tr>
<td>(-1)^n</td>
<td>( u_{n+1} )</td>
<td>( v_n )</td>
</tr>
</tbody>
</table>

Each of the products \( (-1)^{j-1} u_j v_j \) in (1) are obtained by taking the product of \( (-1)^{j-1} \) from the \( j \)th row of the \(+/-\) column, \( u_j \) from the \( j \)th row of the \( u \) column, and \( v_j \) from one row

---

\(^1\)The acronym LIPET (which stands for ‘logarithm, inverse trigonometric, polynomial, exponential, then trigonometric’) helps us choose an appropriate \( u \) in cases where the given integrand is a product of relatively simple functions and other techniques do not suffice. See [10].
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down in the \((j + 1)\)th row of the \(dv\) column (i.e. \(u_j\) and \(v_j\) match up diagonally, as indicated by ‘\(\rightarrow\)’). Summing together yields (1):

\[
\int u \, dv = u_1 v_1 - u_2 v_2 + u_3 v_3 + \cdots + (-1)^n \int v_n \, du_n.
\]

It is important to note that we can stop and derive (1) at any point. The decision to stop is based on the information provided in the last row (which represents the integral \((-1)^n \int v_n \, du_n\)). Also, we may not always want to iterate the IP using \(u_j := u_j' - 1\) and \(v_j := \int v_j \, dx\) for \(j \geq 2\) as above. For instance, it may be preferable to simplify the integrand of some \((-1)^j \int v_j \, du_j\) and apply tabular IP separately to the simplified integral. See Examples 4 and 5 below.

2. Examples and further results

In the remainder of the paper we discuss several examples from calculus and analysis. Further examples can be found in [2], [4], [5], [8], and [10], as well as the references therein. Readers are encouraged to work out the examples themselves by hand using tabular IP and to construct the tables one row at a time.

**Example 1.** Consider \(\int \ln x \, dx\), where \(x > 0\). Let \(u = \ln x\) and \(dv = 1 \, dx\). Note that after generating the second row of the table, we consider the integral of the product of the functions in this row in order to decide what to do next.

<table>
<thead>
<tr>
<th>(+/-)</th>
<th>(u)</th>
<th>(dv)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\ln x)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>((-\frac{1}{x}))</td>
<td>(\rightarrow x)</td>
</tr>
</tbody>
</table>

The second row, which represents \(-\int v \, du\), generates an integral that is easy to compute, so we stop. By (1), we have

\[
\int \ln x \, dx = x \ln x - x + C.
\]

**Example 2.** Consider \(\int e^{3x} \sin 2x \, dx\). Let \(u = e^{3x}\) and \(dv = \sin 2x \, dx\).

<table>
<thead>
<tr>
<th>(+/-)</th>
<th>(u)</th>
<th>(dv)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(e^{3x})</td>
<td>(\sin 2x)</td>
</tr>
<tr>
<td></td>
<td>(9e^{3x})</td>
<td>(\rightarrow -\frac{1}{2} \sin 2x)</td>
</tr>
</tbody>
</table>

At this point, the last row generates a copy of the original integral, so stop and see where things stand. We have

\[
\int e^{3x} \sin 2x \, dx = -\frac{e^{3x}}{2} \cos 2x + \frac{3e^{3x}}{4} \sin 2x - \frac{9}{4} \int e^{3x} \sin 2x \, dx + C.
\]

Adding \(\frac{9}{4} \int e^{3x} \sin 2x \, dx\) to both sides of this equation, then multiplying by \(\frac{4}{13}\) yields

\[
\int e^{3x} \sin 2x \, dx = \frac{e^{3x}}{13}(3 \sin 2x - 2 \cos 2x) + C_0.
\]
Example 3. The tabular approach to IP is especially effective in this example. Consider \( \int (x^2 - 3x) \sin x \, dx \). For the sake of exploration and discovery (in this case, learning from a mistake), let \( u = \sin x \) and \( dv = (x^2 - 3x) \, dx \). This will quickly prove to be a bad choice.

<table>
<thead>
<tr>
<th>+/−</th>
<th>( u )</th>
<th>( dv )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(alt.)</td>
<td>(diff.)</td>
<td>(int.)</td>
</tr>
<tr>
<td>+</td>
<td>( \sin x )</td>
<td>( x^2 - 3x )</td>
</tr>
<tr>
<td>−</td>
<td>( \cos x )</td>
<td>( \frac{x^3}{3} - \frac{3x^2}{2} )</td>
</tr>
</tbody>
</table>

We can immediately see that the integral \( \int \left( \frac{3}{2}x^2 - \frac{1}{3}x^3 \right) \cos x \, dx \) is at least as difficult to handle as the original \( \int (x^2 - 3x) \sin x \, dx \) and does not seem to lead to a solution. Nevertheless, we can apply (1) to obtain

\[
\int (x^2 - 3x) \sin x \, dx = \left( \frac{x^3}{3} - \frac{3x^2}{2} \right) \sin x + \int \left( \frac{3x^2}{2} - \frac{x^3}{3} \right) \cos x \, dx.
\]

While this is technically true, it is certainly not what we want.

Now try again. This time, let \( u = (x^2 - 3x) \) and \( dv = \sin x \, dx \). It may be tempting to fill in the table by columns since our chosen \( u \) is a polynomial and its derivatives eventually become equal to 0. Also, the integrals of \( v_0 \) are easy to compute. Keep in mind, however, that in general we consider our options with the integral generated by each new row, one row at a time.

<table>
<thead>
<tr>
<th>+/−</th>
<th>( u )</th>
<th>( dv )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(alt.)</td>
<td>(diff.)</td>
<td>(int.)</td>
</tr>
<tr>
<td>+</td>
<td>( x^2 - 3x )</td>
<td>( \sin x )</td>
</tr>
<tr>
<td>−</td>
<td>( 2x - 3 )</td>
<td>( -\cos x )</td>
</tr>
<tr>
<td>+</td>
<td>2</td>
<td>( -\sin x )</td>
</tr>
<tr>
<td>−</td>
<td>0</td>
<td>( \cos x )</td>
</tr>
</tbody>
</table>

By (1), we have

\[
\int (x^2 - 3x) \sin x \, dx = (3x - x^2) \cos x + (2x - 3) \sin x + 2 \cos x + C.
\]

In Examples 4 and 5, tabular IP is applied multiple times with multiple tables.

Example 4. Consider \( \int (3x^2 - x) \ln^2 x \, dx \), where \( x > 0 \). Let \( u = \ln^2 x \) and \( v = (3x^2 - x) \, dx \).

<table>
<thead>
<tr>
<th>+/−</th>
<th>( u )</th>
<th>( dv )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(alt.)</td>
<td>(diff.)</td>
<td>(int.)</td>
</tr>
<tr>
<td>+</td>
<td>( \ln^2 x )</td>
<td>( 3x^2 - x )</td>
</tr>
<tr>
<td>−</td>
<td>( \frac{2}{x} \ln x \rightarrow x^3 - \frac{x^2}{2} )</td>
<td>( (−\int (2x^2 - x) \ln x , dx) )</td>
</tr>
</tbody>
</table>
If it is not clear what to do next, stop and assess the situation. Note that the product of $u_2 = (2/x) \ln x$ and $v_1 = x^3 - x^2/2$ (from the second row) simplifies somewhat. So far, we have

$$\int (3x^2 - x) \ln^2 x \, dx = \left( x^3 - \frac{x^2}{2} \right) \ln x - \int (2x^2 - x) \ln x \, dx + C. \quad (2)$$

To evaluate $\int (2x^2 - x) \ln x \, dx$, we apply tabular IP again, but in a separate table.

\[
\begin{array}{c|cc}
+/− & u & dv \\
\hline
+ & \ln x & 2x^2 - x \\
− & \frac{1}{x} & \frac{2x^3}{3} - \frac{x^2}{2} \\
\end{array}
\]

(\int (x/2 - 2x^2/3) \, dx = x^2/4 - 2x^3/9.)

Hence,

$$\int (2x^2 - x) \ln x \, dx = \left( \frac{2x^3}{3} - \frac{x^2}{2} \right) \ln x + \frac{x^2}{4} - \frac{2x^3}{9} + C. \quad (3)$$

Carefully combining (2) and (3) yields

$$\int (3x^2 - x) \ln^2 x \, dx = \left( x^3 - \frac{x^2}{2} \right) \ln x + \left( \frac{x^2}{2} - \frac{2x^3}{3} \right) \ln x + \frac{2x^3}{9} - \frac{x^2}{4} + C_0.$$

Note that our choice of $u$ in the second table is not the derivative of $u_2 = (2/x) \ln x$ from the first table. That is, we did not iterate by setting $u_j := u'_{j−1}$ and $v_j := \int v_{j−1} \, dx$ for $j \geq 2$, but we did apply tabular IP twice.

**Example 5.** In this example tabular IP is applied multiple times. Unlike the previous example, here we iterate by setting $u_j := u'_{j−1}$ and $v_j := \int v_{j−1} \, dx$ for $j \geq 2$, and separate tables are used to compute the successive integrals in the $dv$ column.

Consider $\int xe^x \sin x \, dx$. Let $u = x$ and $dv = e^x \sin x \, dx$. Note that in order to compute $v_1 := \int e^x \sin x \, dx$, we apply IP again; so, for now, we simply write $v_1$ (and $v_2 := \int v_1$) in the $dv$ column.

\[
\begin{array}{c|cc}
+/− & u & dv \\
\hline
+ & x & e^x \sin x \\
− & 1 & v_1 \\
+ & 0 & v_2 \\
\end{array}
\]

(−\int 0 \, dx = C.)

So far, we have

$$\int xe^x \sin x \, dx = xv_1 - v_2 + C, \quad (4)$$

and it remains for us to find $v_1$ and $v_2$. To do this, we use more tables. The computation of
$v_1 = \int e^x \sin x \, dx$ follows just as in Example 2.

<table>
<thead>
<tr>
<th>$+/-$</th>
<th>$u$</th>
<th>$dv$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e^x$</td>
<td>$\sin x$ (The original integrand.)</td>
</tr>
<tr>
<td></td>
<td>$e^x$</td>
<td>$- \cos x$ ($+ \int e^x \cos x , dx$, try another step.)</td>
</tr>
<tr>
<td></td>
<td>$e^x$</td>
<td>$\rightarrow - \sin x$ ($- \int e^x \sin x , dx$, a copy of the original integral.)</td>
</tr>
</tbody>
</table>

The last row generates a copy of the original integral, so we stop and see where things stand. We have

$$v_1 = \int e^x \sin x \, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx.$$  

Adding $\int e^x \sin x \, dx$ to both sides, then dividing both sides by 2 yields

$$v_1 = \int e^x \sin x \, dx = \frac{e^x}{2} (\sin x - \cos x). \quad (5)$$  

Now, note that we have

$$v_2 = v_1 = \int \frac{e^x}{2} (\sin x - \cos x) \, dx = \frac{1}{2} \left( v_1 - \int e^x \cos x \, dx \right). \quad (6)$$  

Another application of IP similar to that just used to find $v_1$ (omitted for the sake of exposition) yields

$$\int e^x \cos x \, dx = \frac{e^x}{2} (\sin x + \cos x). \quad (7)$$  

Therefore, combining (5) and (7) in (6) yields

$$v_2 = \frac{1}{2} \left( v_1 - \int e^x \cos x \, dx \right) = -\frac{1}{2} e^x \cos x. \quad (8)$$  

Finally, carefully combining (4), (5), and (8) yields

$$\int xe^x \sin x \, dx = \frac{e^x}{2} (x \sin x + (1 - x) \cos x) + C. \quad (9)$$  

Remark 1. In Example 5, note that we used the variables $v_1$ and $v_2$ as place holders in the first table. The computation of each of the corresponding integrals merits the use of more space. Regardless of the technique used to compute these integrals (IP, substitution, etc.), the temporary use of extra variables allows us to construct the solution one part at a time by starting with (4) and building up from there.

Other results from calculus and analysis that follow nicely from tabular IP are explored below and in the exercises.

Proposition 1. Let $n$ be a positive integer, and let $x > 0$. Then

$$\int \ln^n x \, dx = x \sum_{k=0}^{n} (-1)^{n-k} \frac{n!}{k!} \ln^k x + C. \quad (9)$$
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Proof. Let $u = \ln^n x$ and $dv = 1 \, dx$.

<table>
<thead>
<tr>
<th>$+/-$</th>
<th>$u$</th>
<th>$dv$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\ln^n x$</td>
<td>$1$</td>
</tr>
<tr>
<td>$+$</td>
<td>$\ln^n x$</td>
<td>$\frac{x}{n}$</td>
</tr>
<tr>
<td>$-$</td>
<td>$n \ln^{n-1} x$</td>
<td>$\to x$</td>
</tr>
</tbody>
</table>

Hence, $\int \ln^n x \, dx = x \ln^n x - n \int \ln^{n-1} x \, dx + C$. Combining this result with an induction argument yields (9).

Theorem 1 provides a statement of Taylor’s formula with integral remainder. Its proof follows readily from tabular IP.

**Theorem 1.** (Taylor’s formula with integral remainder.) If $f : \mathbb{R} \to \mathbb{R}$ has $n + 1$ continuous derivatives on an interval $I$ containing $a$, then, for all $x$ in $I$, we have

$$f(x) = f(a) + f^{(1)}(a)(x-a) + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$+ \int_a^x f^{(n+1)}(t) \frac{(x-t)^n}{n!},$$

where $f^{(j)}$ denotes the $j$th derivative of $f$.

The following proof is essentially identical to that presented in [4].

**Proof of Theorem 1.** By the fundamental theorem of calculus, we have

$$f(x) - f(a) = \int_a^x f^{(1)}(t) \, dt = \int_a^x -f^{(1)}(t)(-1) \, dt.$$

Choose $u = -f^{(1)}(t)$ and, hence, $dv = -1 \, dt$. In the table below, integration and differentiation are performed with respect to $t$, and the first antiderivative of $-1$ is taken to be $x - t$, where $x$ is treated as a constant.

<table>
<thead>
<tr>
<th>$+/-$</th>
<th>$u$</th>
<th>$dv$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-f^{(1)}(t)$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$-$</td>
<td>$-f^{(2)}(t)$</td>
<td>$x - t$</td>
</tr>
<tr>
<td>$+$</td>
<td>$-f^{(3)}(t)$</td>
<td>$-(x-t)^2$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$(-1)^{n-1}$</td>
<td>$-f^{(n)}(t)$</td>
<td>$(-1)^{n-1} \frac{(x-t)^{n-1}}{(n-1)!}$</td>
</tr>
<tr>
<td>$(-1)^n$</td>
<td>$-f^{(n+1)}(t)$</td>
<td>$(-1)^{n+1} \frac{(x-t)^n}{n!}$</td>
</tr>
</tbody>
</table>
Therefore,
\[ f(x) - f(a) = \int_a^x f'(t)(-1) \, dt \]
\[ = \left[ -f^{(1)}(t)(x-t) - \frac{f^{(2)}(t)}{2!}(x-t)^2 - \cdots - \frac{f^{(n)}(t)}{n!}(x-t)^n \right]_a^x \]
\[ + \int_a^x \frac{f^{(n+1)}(t)}{n!}(x-t)^n \, dt. \]
The result follows immediately. \( \square \)

3. Closing remarks and further results

Tabular IP can be used to determine a wide variety of integrals (see [2], [4], [5], [8], and [10]). For a polynomial \( P(x) \) and constants \( a \neq 0, b \neq 0, \) and \( q \neq 0, 1, \) integrals of the form
\[ \int P(x) (ax + b)^q \, dx \]
are readily computed using tabular IP since successive derivatives of \( P(x) \) eventually vanish (see [4]).

As in Example 3, it is especially convenient when 0 appears in the \( u \) column. This happens whenever \( u \) is chosen to be a polynomial, and we iterate by setting \( u_j := u_{j-1} \) and \( v_j := \int v_{j-1} \, dx \) for \( j \geq 2. \) However, it is not necessary for these conditions to hold in order to know when to stop or for tabular IP to be effective. Rather, with tabular IP, we proceed by considering our options with the integral \((-1)^j \int v_j u_j \) generated by each new row, one row at a time.

The tabular method also allows us to compute integrals of the form \( \int \sin ax \cos bx \, dx, \)
\( \int \sin ax \sin bx \, dx, \) and \( \int \cos ax \cos bx \, dx \) where \( a \neq b, a \neq 0, \) and \( b \neq 0 \) without resorting to the use of trigonometric identities. Similarly, integrals like \( \int \sin^2 ax \, dx \) for nonzero \( a \) also follow from IP after an application of the Pythagorean identity \( \sin^2 ax = 1 - \cos^2 ax. \) See [8].

Given a function \( f : [0, \infty) \to \mathbb{R}, \) its Laplace transform
\[ \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) \, dt \]
is often determined by an application of IP. Indeed, the integrand is a product, and it is easy to both integrate and differentiate \( e^{-st}. \) Similarly, Laplace transform formulae, such as that for the \( n \)th derivative of a function, follow in an especially nice way from tabular IP. See [4] for details and further results in calculus and analysis that follow readily from tabular IP, including a proof of the residue theorem. See [5] for a natural connection between IP and the development of certain infinite series.

An interesting alternative form of tabular IP can be found in [3]. In that paper, the tables are generated with a straightforward format which may require more writing than those described here and in [2], [4], [5], [8], and [10]. In [11], a type of IP formula based on the quotient rule for derivatives is developed and explored.

4. Exercises

**Exercise 1.** Evaluate \( \int x^n \sin ax \, dx, \) where \( n \in \mathbb{N} \) and \( a \neq 0. \)

**Exercise 2.** Evaluate \( \int x^2 e^x \sin x \, dx. \)
Exercise 3. Evaluate \( \int \ln(x^2 + 4x + 7) \, dx \).

Exercise 4. Show that \( nb \int_0^1 (1 - s)^n s^{b-1} \, ds = n! n^b / (b(b + 1) \cdots (b + n)) \), where \( n \in \mathbb{N} \) and \( b > 0 \). (This integral is used to study the gamma function in [4, p. 418].)

Exercise 5. Show that, for \( x \geq 0 \) and any positive integer \( n \),

\[
f(x) = \int_x^\infty t^{-1} e^{x-t} \, dt = \frac{1}{x} - \frac{1}{x^2} + \frac{2!}{x^3} - \cdots + (-1)^{n-1} \frac{(n-1)!}{x^n} + (-1)^n n! \int_x^\infty t^{-n-1} e^{x-t} \, dt.
\]

(See [7, Example 7.2.2].)

References


