SOME ASPECTS OF GAMBLING WITH THE KELLY CRITERION

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Abstract
In this paper we consider the problem of gambling with the Kelly criterion, i.e. gambling so as to maximize the expected exponential rate of capital growth. We consider gambling on games of chance such as horse races, as well as gambling involved in the buying and selling of shares on the stock market. For both these situations we obtain results which in some way are surprising and run counter to intuition.

Keywords: Gambler’s ruin; Kelly criterion; hedging; stock market; exponential growth

2010 Mathematics Subject Classification: Primary 91A60
Secondary 60G40

1. Introduction
In Phatarfod (1999) the author considered betting strategies in horse races in the framework of the gambler’s ruin problem. That is, we have a gambler with a finite capital, who bets for a fixed positive expected gain in each race, and we are interested in the probability of him losing his capital. It was shown that a betting strategy which intuition would suggest as being optimal was, in fact, not so. For example, suppose that in a race there are, among others, three horses, A, B, and C, with odds on offer against their winning the race being 5/1, 3/1, and 3/1 respectively and the probabilities of their winning the race being 0.2, 0.3, and 0.3 respectively. Betting on horse A alone with its high odds and expectation of gain of 0.2 per unit stake seems intuitively appealing. However, with a capital of $80, and betting $8 per race, the probability of the gambler’s ruin is 0.466 2, whereas betting amounts $2, $3, and $3 on horses A, B, and C respectively (to give the same expectation of $1.60 of gain on the race) would dramatically reduce this probability to 0.000 13. This is because the variance of the net gain in the first case is 368.64, whereas in the second case it is 23.04. An approximate relation between ruin probability and variance is

\[ P(\text{ruin}) \approx \exp \left( -\frac{2\mu a}{\sigma^2} \right), \]

where \( \mu \) and \( \sigma^2 \), respectively, are the mean and variance of the net gain, and \( a \) is the gambler’s capital. The importance of the reduction of variance is so crucial that, for example, in some situations, betting on a favourable bet (expected gain positive) alone is not as advantageous as having an additional fair bet (expected gain zero) and, in some circumstances, having an unfavourable bet (expected gain negative); thus, in terms of reducing the probability of ruin it is better to dilute the favourableness of the original favourable bet with a fair or an unfavourable bet. This is because this dilution reduces the variance of the net gain for each race.

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In this paper we consider gambling in the framework of the Kelly criterion; see e.g. Rotando and Thorp (1992) and Breiman (1961). This paper essentially follows the discussion in Rotando and Thorp (1992), by considering the importance of the variance of the net gain. We have a gambler with a finite capital who wants to bet in such a way as to maximize the expected exponential rate of growth of his capital, i.e. the Kelly criterion. This is achieved by wagering an optimal fraction of his capital at each stage. As in Rotando and Thorp (1992), we consider games of chance in general, (binomial games), not necessarily horse races, as well as gambling involved in the buying and selling of shares on the stock market. In the latter case, it is convenient to consider the net gain as a continuous random variable. Another difference from the binomial games is that the time period per game is much greater, typically a year. The purpose of this paper is to show that here too the results obtained run counter to intuition. There are two types of results which fall into this category. For example, suppose that the gambler wins a units for each unit wagered when he wins, and suppose that the win probability is \( p > 0 \), so that the game is favourable with mean gain \( \mu = ap - q > 0 \), where \( q = 1 - p \). Rotando and Thorp (1992) have shown that the optimal fraction to bet at each stage is

\[
f^* = \frac{ap - q}{a}.
\]

Suppose that the gambler’s capital is $1000. Then, for the case \( p = 0.9, q = 0.1, \) and \( a = 0.2 \), we have \( \mu = 0.08 \), and the expectation of capital at the end of 100 games, using the optimal fraction \( f^* \), is 23,332.95, whereas for the case \( p = 0.54, q = 0.46, \) and \( a = 1 \), the value of \( \mu \) is 0.08 as before, but the expectation of the capital at the end of 100 games is a mere 1,892.62. This is so because the optimal fraction of the capital wagered for the former case is 0.4, while for the latter it is 0.08; these values are directly dependent on the value of \( a \), which also influences the value of the variance.

Now, consider again the case, \( p = 0.54, q = 0.46, \) and \( a = 1 \). Here, the gambler’s edge (i.e. the mean gain per unit stake) is 0.08, i.e. a somewhat substantial one; the fraction wagered is \( f^* = 0.08 \), a fairly conservative value. So we expect a steady increase of the capital at an exponential rate to the unspectacular value of 1,892.62. However, simulation results differed quite substantially from the ‘expected values’. Half of the simulations gave final capitals below $1,000, with the gambler losing on the exercise. In fact, there is a significant probability of such an event occurring. Of course, we know that eventually the final capital must increase indefinitely. However, in many practical situations we are interested not in the ultimate future, but in a finite time horizon. The reason we have a significant probability of losing in a finite time horizon is that, although the expectation of the capital increases exponentially, so does the standard deviation, which in fact increases faster than the expectation.

For the cases of binomial games as well as continuous variable games, we consider a sequence of 100 games. The number 100 has been arbitrarily chosen, but is of the same order of magnitude as the numbers chosen by other authors; see Wong (1981). For games of chance this may well be a small value, but for stock market gambles the value is rather high. For both cases, the problem of having a substantial probability of being on the losing side at the end of the trials can be somewhat alleviated by diversification, i.e. having more than one bet at the same time. This may be somewhat difficult to carry out in the horse race situation, unless we have the resources to bet at many venues at the same time.

The results in the stock market case can be extended to the situation concerning a commercial company’s operations and, in particular, its bankruptcy, an area which was, in fact, the main motivation behind the present study of gambling with the Kelly criterion. A commercial
company’s operations can be likened to those of a stock market investor. The latter invests (i.e. gambles with) a fraction \( f \) of the capital available to him, leaving the rest uninvested. Similarly, a commercial company invests a fraction of its capital in ventures which may be somewhat risky, and are effectively gambles. Of course, not to invest in this way may not be the best course of action, as such investments may well bring substantial returns. The important thing is to arrive at the correct value of the fraction of money so invested. There are many reasons why commercial companies fail, but surely one of the reasons is that the company overextends itself, i.e. in the context of the present theory, has a larger value of \( f \) than is desirable. In Section 3 we show the relation between the parameters of the underlying distribution and the value of the optimal fraction. It is suggested that a company wishing to minimize the probability of bankruptcy would need to estimate the parameters very carefully so as to arrive at the correct value of the optimal fraction.

2. Binomial games

Let \( Y \) be the net gain made by a gambler on a game with unit stake and let \( E(Y) = \mu > 0 \). Let us assume that we have a sequence of games, for each of which the net gain per unit stake has a distribution the same as that of \( Y \). Let the capital at the end of game \( n \) be \( X_n \). Taking the simple case when the probability of winning a unit amount is \( p \) and the probability of losing the bet is \( q \) (where \( p + q = 1 \)), we have, with \( X_0 \) as the initial capital and where we are wagering a fraction \( f \) of the capital at each stage,

\[
X_n = X_0(1 + f)^S(1 - f)^F,
\]

where \( S \) and \( F \), respectively, are the number of successes and failures in \( n \) trials (i.e. \( S + F = n \)). To make it possible to wager a fraction at each stage, we assume that the capital is infinitely divisible. Taking \( 0 < f < 1 \), we have \( P(X_n = 0) = 0 \); thus ruin in the sense of the gambler’s ruin problem does not occur. If ruin is interpreted to mean that, for any arbitrary small positive \( \epsilon \),

\[
\lim_{n \to \infty} P(X_n \leq \epsilon) = 1,
\]

then ruin is possible if the value of \( f \) is large. Now, since

\[
\exp(\frac{1}{n} \log(\frac{X_n}{X_0})) = \frac{X_n}{X_0},
\]

the quantity

\[
\frac{1}{n} \log(\frac{X_n}{X_0}) = \frac{S}{n} \log(1 + f) + \frac{F}{n} \log(1 - f)
\]

measures the exponential rate of growth per trial. The Kelly criterion maximizes the expected value of this growth, namely

\[
G(f) = E\left(\frac{\log(X_n/X_0)}{n}\right) = p \log(1 + f) + q \log(1 - f).
\]

Setting \( G'(f) = 0 \), we have the optimal fraction \( f^* \), as \( f^* = p - q \). The behaviour of \( G(f) \) is illustrated in Figure 1, where \( f^c \) (\( f^c \neq 0 \)) is such that \( G(f^c) = 0 \).

Table 1 gives values of \( f^* \) and \( f^c \) for a selection of values of \( \mu \). We see that \( f^c \geq 2f^* \). We can show that this is generally true by noting that \( G(f) \) is approximately symmetrical about \( f^* \).
To show this let \( x = f - (p - q) \). Using a Taylor series expansion for \( H(x) = G(x + p - q) \), we have
\[
H(x) \cong \log 2 + p \log p + q \log q - \frac{x^2}{8pq} + \frac{x^3(q - p)}{24p^2q^2}.
\]
For example, for \( p = 0.53 \) and \( q = 0.47 \) we have
\[
H(x) \cong 0.6949 - 0.5018x^2 + 0.0403x^3,
\]
which is a very nearly symmetrical function.

When \( f < f^c \) (\( f \neq f^* \)), the expected value of capital increases exponentially, but at a slower rate than with \( f^* \). For \( f > f^c \), ruin is certain. Breiman (1961) showed that

(a) If \( G(f) > 0 \) then \( \lim_{n \to \infty} X_n = \infty \) almost surely.

(b) If \( G(f) < 0 \) then \( \lim_{n \to \infty} X_n = 0 \) almost surely.

It is easy to work out the mean and variance of \( X_n \). When \( f = f^* \), we obtain
\[
X_n = X_{n-1}(1 + f^*) \quad \text{with probability } p
\]
\[
= X_{n-1}(1 - f^*) \quad \text{with probability } q.
\]
Thus,
\[
E(X_n \mid X_{n-1}) = X_{n-1}(1 + f^*(p - q))
\]

**Table 1:** Values of \( f^*, f^c, E(X_n), \sigma(X_n) \), and \( P^* = P(X_n < X_0) \), for different values of the mean net gain \( \mu \).

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( f^* )</th>
<th>( f^c )</th>
<th>( E(X_n) )</th>
<th>( \sigma(X_n) )</th>
<th>( P^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p = 0.53, ; q = 0.47 )</td>
<td>0.06</td>
<td>0.06</td>
<td>0.1197</td>
<td>1432.40</td>
<td>935.89</td>
</tr>
<tr>
<td>( p = 0.54, ; q = 0.46 )</td>
<td>0.08</td>
<td>0.08</td>
<td>0.1593</td>
<td>1892.62</td>
<td>1765.20</td>
</tr>
<tr>
<td>( p = 0.6, ; q = 0.4 )</td>
<td>0.20</td>
<td>0.20</td>
<td>0.3894</td>
<td>50504.95</td>
<td>284555</td>
</tr>
</tbody>
</table>
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Table 2: Values of $E(X_n)$, $\sigma(X_n)$, and $P^* = P(X_n < X_0)$, for different values of $a$ where $\mu = 0.08$.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$f^*$</th>
<th>$E(X_n)$</th>
<th>$\sigma(X_n)$</th>
<th>$P^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2, $p = 0.9$</td>
<td>0.4</td>
<td>23 332.95</td>
<td>56 568.4</td>
<td>0.1513</td>
</tr>
<tr>
<td>0.5, $p = 0.9$</td>
<td>0.16</td>
<td>3 567.54</td>
<td>5 148.08</td>
<td>0.2796</td>
</tr>
<tr>
<td>1, $p = 0.54$</td>
<td>0.08</td>
<td>1 892.62</td>
<td>1 763.20</td>
<td>0.3446</td>
</tr>
<tr>
<td>1.5, $p = 0.432$</td>
<td>0.053</td>
<td>1 530.75</td>
<td>1 124.35</td>
<td>0.3730</td>
</tr>
<tr>
<td>2, $p = 0.36$</td>
<td>0.04</td>
<td>1 376.42</td>
<td>839.30</td>
<td>0.3897</td>
</tr>
<tr>
<td>10, $p = 0.098$</td>
<td>0.008</td>
<td>1 066.07</td>
<td>283.77</td>
<td>0.4509</td>
</tr>
</tbody>
</table>

Table 3: Simulated values of $X_{100}$ for the case $X_0 = 1 000$, $p = 0.54$, and $q = 0.46$.

<table>
<thead>
<tr>
<th></th>
<th>$E(X_n)$</th>
<th>$\sigma(X_n)$</th>
<th>$P^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>646.32</td>
<td>783.44</td>
<td>872.00</td>
<td>1 197.00</td>
</tr>
<tr>
<td>3 427.00</td>
<td>628.49</td>
<td>1 060.93</td>
<td>665.00</td>
</tr>
<tr>
<td>853.05</td>
<td>4 656.26</td>
<td>341.93</td>
<td>696.81</td>
</tr>
<tr>
<td>2 080.44</td>
<td>1 110.93</td>
<td>595.67</td>
<td>1 268.01</td>
</tr>
</tbody>
</table>

and, hence,

$$E(X_n) = X_0 K^n,$$

where $K = 1 + f^*(p - q)$. Similarly,

$$\sigma^2(X_n) = X_0^2 L^n,$$

where $L = 1 + (f^*)^2 + 2 f^*(p - q)$. Thus,

$$\sigma^2(X_n) = X_0^2 (L^n - K^{2n}).$$

Table 1 also gives the values of $E(X_n)$ and $\sigma(X_n)$, for $n = 100$ and $X_0 = 1 000$. Basically, as $\mu$ increases, the fraction wagered, $f^*$, increases, resulting in an increase in the expected value of the final capital $X_{100}$. However, $\sigma(X_n)$ increases much faster than $E(X_n)$.

We obtain similar results when the gambler wins $a$ units for every unit wagered. Here, for $\mu = ap - q$ and $f^* = (ap - q)/a$, we obtain

$$E(X_n) = X_0(1 + (ap - q) f^* )^n,$$

$$\sigma^2(X_n) = X_0^2((1 + (f^*)^2(pa^2 + q) + 2 f^*(pa - q))^n - (1 + f^*(pa - q))^{2n}).$$

Table 2 gives values of $E(X_n)$ and $\sigma(X_n)$ for different values of $a$ (for the same mean, $\mu = 0.08$). These show what effect the value of $a$, and indirectly the variance of the net gain, has on the expected value and the standard deviation of $X_n$. The expected values range from 23 332.95 to 1 066.07. It can be shown by some straightforward but somewhat tedious algebra that, for a fixed $\mu$, as $a$ increases, $f^*$ decreases, resulting in decreasing values of $E(X_n)$ and $\sigma(X_n)$.

The preceding discussion leads us to expect that if the gambler sticks to the optimal fraction $f^*$, a steady increase of his capital at an exponential rate is guaranteed, particularly so for the case in which $p = 0.54$ and $q = 0.46$. Here, the gambler’s edge is 0.08, i.e. somewhat substantial, and the fraction of the capital wagered is $f^* = 0.08$, a fairly conservative value. However, the results are quite different from what we would expect. Table 3 gives the results of 20 simulations for this case with $n = 100$ and $X_0 = 1 000$. Half of the final capital values are
smaller than the initial capital of $1000. Of course, we know that eventually $X_n$ must increase indefinitely. However, in many practical situations we are interested not in the ultimate future but in a finite time horizon.

To derive the probability $P^* = P(X_n < X_0)$ that the gambler would be on the losing side at the $n$th stage, we consider the equivalent probability $P(\log X_n < \log X_0)$. To derive the latter, for the case of a general $a$, we have

$$\log X_n = \log X_0 + S \log(1 + af^*) + F \log(1 - f^*),$$

so that

$$E(\log X_n) = \log X_0 + c,$$

$$\sigma(\log X_n) = d,$$

where

$$c = n\left(p \log \left(\frac{1 + af^*}{1 - f^*}\right) + \log(1 - f^*)\right)$$

and

$$d = \sqrt{npq \log \left(\frac{1 + af^*}{1 - f^*}\right)}.$$

For large $n$, $\log X_n$ is approximately normal; hence,

$$P^* = P \left( Z < -\frac{c}{d} \right),$$

where $Z$ is a standardized normal variable.

Tables 1 and 2 give the values of $P^*$ for different values of $\mu$ with $a = 1$, and for different values of $a$ for $\mu = 0.08$. It can be shown that, for a constant value of $\mu$, as $a$ increases and the variance of the net gain increases, $f^*$, $E(\log X_n)$, and $\sigma(\log X_n)$ decrease. However, $P^*$ increases.

As in Phatarfod (1999), here it is also advantageous to dilute a favourable bet by having an additional fair bet, since such a dilution reduces the variance of the net gain, keeping the mean the same. Suppose that we have bets of equal amounts on horse A with odds of $a/1$ and probability of winning equal to $p_1 = (1 + \mu)/(1 + a)$, and a fair bet at even money on horse B, whose probability of winning is 0.5, the total amount determined by the fraction $f$ of the capital at that stage. When horse A wins, the net gain for a unit amount is $a - 1$; when horse B wins, there is no net gain or loss; when neither horse A nor horse B wins, there is a loss of two units. Let $S$ be the number of times horse A wins, and $F$ be the number of times neither horse A nor horse B wins. Then we have

$$X_n = X_0 \left(1 + \frac{fa}{2}\right)^S (1 - f)^F,$$

from which we obtain, in an obvious manner,

$$f^* = \frac{\mu}{(a - 1)(p_1 + q)}, \quad \text{where} \quad p_1 + 0.5 + q = 1.$$
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Table 4: Values of $f^*$, $E(X_n)$, $\sigma(X_n)$, and $P^* = P(X_n < X_0)$, for different values of $a$ where $\mu = 0.08$, and when an additional fair bet at even money is taken.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$f^*$</th>
<th>$E(X_n)$</th>
<th>$\sigma(X_n)$</th>
<th>$P^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.32</td>
<td>3567.54</td>
<td>4412.28</td>
<td>0.2660</td>
</tr>
<tr>
<td>2</td>
<td>0.16</td>
<td>1892.62</td>
<td>1669.64</td>
<td>0.3228</td>
</tr>
<tr>
<td>10</td>
<td>0.0177</td>
<td>1073.65</td>
<td>300.32</td>
<td>0.4483</td>
</tr>
</tbody>
</table>

3. Continuous games

We now consider the case of an investor involved in buying and selling shares on the stock market. For simplicity, we assume that the share price movement is uniformly distributed. Suppose that the net gain per unit investment is a continuous random variable $U$ uniformly distributed over the interval $(a, b)$, $-1 < a < 0, 0 < b < 1$. Translated into the stock market situation, an investor purchases a stock for $100 per share, and the anticipated price per share in one year’s time is uniformly distributed over $(100(1 + a), 100(1 + b))$. For example, for $a = -0.7$ and $b = 1$, the current share price is $100, and the anticipated price is uniform over (30, 200). As before, we have the growth coefficient

$$G(f) = \int_a^b \frac{\log(1 + fu)}{b - a} \, du.$$  

Setting $G'(f) = 0$ gives us $f^*$, the optimal fraction of the capital wagered (i.e. invested) at each stage, as the positive solution to

$$(b - a)f = \log\left(\frac{1 + bf}{1 + af}\right).$$

To determine the expected value and the variance of the capital at the $n$th stage, using the optimal value of $f$, we have

$$X_n = X_{n-1}(1 - f^*) + X_{n-1}f^*(1 + U).$$

This is because the capital at the $n$th stage is the sum of the capital not invested at the $(n-1)$th stage, namely $X_{n-1}(1 - f^*)$, and the return on the investment of $X_{n-1}f^*$; we assume, for simplicity, that the capital not invested remains idle during the year, without attracting interest. From the above we obtain

$$X_n = X_{n-1}(1 + fU).$$

Working in a similar manner to that in Section 2, and using $E(U) = (a + b)/2$ and $\sigma^2(U) = (b - a)^2/12$, we obtain

$$E(X_n) = X_0K^n,$$

$$\sigma^2(X_n) = X_0^2\left(K^2 + \frac{(f^*)^2(b - a)^2}{12}\right)^n - K^{2n},$$

where $K = 1 + f^*(a + b)/2$.

Note that we sometimes have $f^* > 1$. For example, when $a = -0.3$ and $b = 0.5$ we obtain $f^* = 1.95$. When this happens, adopting the optimal policy involves borrowing an
Table 5: The mean of $X_n$, the standard deviation of $X_n$, and $P^* = P(X_n < X_0)$, for cases A, B, C, and D.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$\mu$</th>
<th>$\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>case A</td>
<td>-0.7</td>
<td>1.0</td>
<td>0.15</td>
<td>0.2408</td>
</tr>
<tr>
<td>case B</td>
<td>-0.35</td>
<td>0.5</td>
<td>0.075</td>
<td>0.0602</td>
</tr>
<tr>
<td>case C</td>
<td>-0.3</td>
<td>0.5</td>
<td>0.10</td>
<td>0.0533</td>
</tr>
<tr>
<td>case D</td>
<td>-0.84</td>
<td>1.0</td>
<td>0.08</td>
<td>0.2821</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$f^*$</th>
<th>$E(X_{100})$</th>
<th>$\sigma(X_{100})$</th>
<th>$P^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>case A</td>
<td>0.63</td>
<td>8347887</td>
<td>387709456</td>
<td>0.0582</td>
</tr>
<tr>
<td>case B</td>
<td>1.26</td>
<td>8347887</td>
<td>387709456</td>
<td>0.0582</td>
</tr>
<tr>
<td>case C</td>
<td>1.95</td>
<td>54459469127.7</td>
<td>41722243155.4</td>
<td>0.0122</td>
</tr>
<tr>
<td>case D</td>
<td>0.28485</td>
<td>9518.37</td>
<td>26435.60</td>
<td>0.2246</td>
</tr>
</tbody>
</table>

amount, $(f^* - 1)X_n$, at the $n$th stage to invest the amount $f^*X_n$. We assume that out of the proceeds of the investment, the amount $(f^* - 1)X_n$ is returned, with no interest charged. Thus,

$$X_n = X_{n-1}f^*(1 + U) - X_{n-1}(f^* - 1)$$

$$= X_{n-1}(1 + f^*U),$$

as before.

Table 5 gives the expected values and $\sigma(X_{100})$ when $X_0 = $1 000. We see that the expected values are not only phenomenally large, but are widely disparate; so are the standard deviations. Note that case B differs from case A by only a scale change, and so the values regarding the final capital remain the same. Also, case C has smaller mean net gain per investment than case A; however, the standard deviation is also small. This results in a large value of $f^*$ and, hence, phenomenally large values of $E(X_{100})$ and $\sigma(X_{100})$. Case D can be compared to an example of the binomial case, namely the equivalent binomial gamble case, i.e. with $\mu = 0.08$, $\sigma^2 = 0.2821$, $p = 0.8052$, and $a = 0.3412$. This has $f^* = 0.2345$. For this case, $E(X_{100}) = 6414.87$ and $\sigma(X_{100}) = 11845.37$. The higher values for these in case D is attributable to the fact that the value of $f^*$ is higher, 0.28485 compared to 0.2345.

We shall now see that, in spite of the phenomenal growth of the capital, there is a significant probability that at stage 100 the capital has fallen below the original value.

It is easy to work out the mean and variance of $\log(X_n/X_0)$. We have

$$\log\left(\frac{X_n}{X_0}\right) = \sum \log(1 + f^*U_i),$$

where $U_i$ is uniformly distributed over $(a, b)$. This gives

$$E\left(\log\left(\frac{X_n}{X_0}\right)\right) = nA,$$

$$\sigma^2\left(\log\left(\frac{X_n}{X_0}\right)\right) = nB,$$
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where

$$A = \frac{(1 + bf^*) \log(1 + bf^*) - (1 + af^*) \log(1 + af^*) - (b - a) f^*}{f^*(b - a)},$$

$$B = \frac{(1 + bf^*) \log(1 + bf^*)^2 - (1 + af^*) \log(1 + af^*)^2}{f^*(b - a)} - 2A - A^2.$$

We find that

$$P^* = P(X_n < X_0)$$

$$= P\left(\log X_n < \log X_0\right)$$

$$= P\left(Z < -A \sqrt{\frac{n}{B}}\right).$$

Table 5 summarizes the situation for the various cases.

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References


