MARKOV AND LIFE INSURANCE

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Abstract

This article outlines Markov’s involvement with life insurance, particularly in the areas of retirement funds and juvenile insurance.

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1. Introduction

Andrei Andreevich Markov’s (1856–1922) life and work in the theory of probability are well known (see Seneta (2001) for further references), but his interest in life insurance has not been previously studied. The following information appeared in Russian in Istoriko-Matematicheskie Issledovania, 1997, Vol. 2 (37), pp. 22–33 and was privately printed in 50 copies in English (see Sheynin (2004, pp. 210–221)).

In 1906 Markov published two polemic articles (letters) on the insurance of children. Later he (see Section 4) referred to them without mentioning his authorship; it is therefore no wonder that Alekseeva (1951) did not mention them. Neither did Grodzensky (1987) cite them, although it was from him that I first came to know of their existence. (Grodzensky reprinted many letters on burning public issues sent by Markov to various newspapers, some of which, obviously rejected, he had found in archives.) I comment on these articles in Section 4 and reprint them in Section 5, and in Section 3 I describe Markov’s activities in insurance; Section 2, that discusses the history of insurance, contains material which is largely known. There, I draw, among other sources, on my previous paper (Sheynin (1977)); in addition Kohli and van der Waerden (1975) and Hald (1990) devoted much attention to this subject.

I define life insurance as any agreement ensuring payments of definite sums either to the heir(s) of the insured should he/she die within a stipulated period of time (a lump sum), or to the insured himself (regular sums and, especially, an annuity). According to modern ideas, though not in line with the practice of insurance during the 17th and 18th centuries, the price of such agreements is to be determined by means of mortality tables depending on the age and the sex of the insured. My definition does not cover all existing forms of life insurance (see Section 4), but it is sufficient for a general understanding of the topic. I also note that various kinds of mutual insurance of several persons have also been widely used. Thus, upon paying a certain sum, a married couple could enjoy a fixed annuity until one of them died, with the surviving spouse continuing to draw it to the end of his/her life.

In England, societies offering mutual insurance were already in existence in the 17th century. At the turn of the next century England had several thousand of them. Their members drew

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insurance in cases of illness or death of their wives, and wives received a sum upon the death of their husbands. It seems (see Wells (1965)) that most such societies existed on the basis of voluntary dues, but that there had been no connection between the premiums and the ages of their members.

An operation connected with risk is called fair if the expected gain $\xi$ is zero (i.e. $E \xi = 0$). For an insured individual, insurance is never fair: since insurance societies cannot exist without profit, the individual’s expectation is always negative. Nevertheless, insurance may be advantageous for the insured, for example if a family gets a sum of money should its breadwinner die prematurely. And, indeed, such scientists as Laplace (1812, p. 454) ardently approved of the institution of life insurance. In 1898, more than 7 million people were insured the world over, about 0.1 million of them in Russia (Press (1901, p. 747)); this indicates the scale of the activities of the main insurance enterprises at the end of the 19th century.

2. From the history of life insurance

Population statistics was the most important branch of political arithmetic that emerged in the mid-17th century; at least until the beginning of the 19th century such significance persisted mainly because the developing insurance business demanded reliable data on mortality, and studies of its laws. These statistical data, insofar as they were being collected by insurance societies, had been kept secret, but the theoretical principles were not concealed. Their development both directly and implicitly heightened the interest in probability, and to some extent fostered its advance.

In a letter to his brother Lodewijk devoted to various problems of mortality, Huygens (1669) calculated the expectations of the order statistics for an empirical distribution, introduced the concepts of mean and probable durations of life and constructed and made methodological use of a graph of a continuous function

$$y = 1 - F(x),$$

where $F(x)$ represented an integral distribution function of mortality. It was in this correspondence that the theory of probability went beyond the province of games of chance (as it also had in 1671 at the hands of De Witt).

Niklaus Bernoulli (1709) considered a number of problems connected with insurance. In one of these he (see Bernoulli (1709, pp. 296–297), also see Todhunter (1865, pp. 195–196)) determined the expectation of the maximal element of a sample from a continuous uniform distribution. Using statistical data published by Halley in 1694, De Moivre (1725) proposed a uniform distribution of mortality, beginning with age 12. There he also introduced the expectation of a random variable with this distribution (see Problem 20 from part 1 of De Moivre (1725)) and calculated probabilities of the type

$$P(\xi \geq x) = 1 - F(x)$$

for it (see Chapter 8 of part 2 of De Moivre (1725), in my own notation).

Laplace (1812, Chapter 9) solved several problems of life insurance in the same way as problems in the treatment of observations, but this time he also discussed the ‘Poisson’ generalization of the Bernoulli trials. Gauss did not shun life insurance either; he had to solve practical problems while managing the pension fund at Göttingen University (see Sheynin (1979, pp. 61–64)). In Russia, Zernov (1843) published a treatise in which he paid special attention to life insurance, and Buniakovsky (1846) devoted a chapter of his celebrated work to the same subject.
3. Markov’s work on retirement funds

Retirement funds began appearing in Russia in the second half of the 19th century. After retirement, members of these funds drew lifelong pensions depending on the duration of their work and their final or mean salary. It was indeed possible to estimate the duration of life of the pensioners by applying mortality tables (although, strictly speaking, statistical inferences suitable for the general population will not do for any special group), but it was extremely difficult to predict the yearly number of those retiring, or their salaries, while the evaluation of the number of additional members of a given fund admitted for state reasons (see below) was absolutely impossible. Given that the widows and children of dying members were also provided with life annuities or long-term pensions, it becomes evident that any retirement fund could easily go broke, and especially so during the first years of its existence, when experience was still lacking and unreliable guesswork was necessary.

Ostrogradsky (1858) participated in the work of the first Russian retirement fund. A few decades later Markov began to busy himself with similar activities; already in 1884 he (see Markov (1884)) published detailed calculations for the retirement fund at the Ministry of Justice. In 1890 he became a member of its governing board (see Anonymous (1890–1891, Vol. 2, p. 36)). He actively participated in its sittings, offered his advice about concrete issues and checked bookkeeping accounts. Thus, he (see Markov (1891)) compiled a note (not mentioned by Alekseeva (1951)) on the financial conditions necessary for ensuring the payment of pensions. Volume 1 of the same source contains many references to Markov, and on pages 90 and 100 it cites pages 10 and 6 respectively of a certain note, possibly due to Markov (1884), since his other note is only two pages long.

In 1893, 1894 and 1902 Markov received letters of thanks from the Ministry of Justice (Grodzensky (1987, p. 59)). Incidentally, its retirement fund was considered the ‘best established’ from among the six funds of the ‘civil departments’. This fact was attributed to Markov’s ‘precise mathematical calculations’ (Lykoshin (1886)), but it may have been more correct to mention his prudence and foresight, and perhaps his intuition and ability to detect the slightest change in conditions.

Markov (1899) also occupied himself with similar work at the War Ministry. In 1900, academician Sonin (Anonymous (1900a)), on behalf of the Physical and Mathematical Department of the Academy of Sciences, acquainted himself with the work of the Ministry’s retirement fund and expressed the following opinion about it: ‘the sole reason for the crisis that it experiences now’ was the unforeseeable increase in the number of its members occurring through instructions from above. He recommended the liquidation of the fund and the transfer of its liabilities to the state.

After hearing this out, Markov (see Anonymous (1900a)) declared that he did not agree with Sonin ‘on any point’. The Department resolved that, since the problem posed by the War Ministry (before the Academy) was of a practical rather than of a purely scientific nature, it should only inform the Ministry that ‘the members of the Academy are always ready to render assistance’ to it.

Also in 1900 the same Ministry established a ‘Special (standing) Mathematical Conference’ to determine the financial state of its fund (Anonymous (1900b, p. 10)). Its members included academicians Markov, Sonin and I. I. Yanzhul, other eminent scientists I. I. Pomerantsev and N. Ya. Tsinger, and actuaries including B. F. Maleshevsky. Regrettably, nothing is known either about the work of this Conference, or of Markov’s even more active participation in practical life insurance after his retirement in 1906 (Markov, Jr. (1951, p. 604)).
Again in 1900 and later, Markov devoted a short chapter of his textbook (see Markov (1908)) to life insurance. There, not aiming at new results, he acquainted his readers with the main stochastic problems of the contemporary insurance business. I indicate, finally, that the Markov Fond (Inventory 1 of Fond 173) at the Archive of the Russian Academy of Sciences includes three letters directly pertaining to this subject (Delo 60, No. 15 and Delo 5, No. 5 and 7). In the first of these, in 1900, Markov indicated that he ‘once again (!) had turned his attention to the theory of inability to work’ and discussed the mortality of the disabled. The two other letters dated 1916 were written to him by Dmitry Aleksandrovich Grave (1863–1939) and testified that Markov had been busying himself, both theoretically and practically, with the work of pension funds.

4. Markov’s newspaper publications

In Section 5 I will reprint two polemic newspaper articles published by Markov and devoted to the insurance of children. It seems that this kind of insurance was fairly widely practised in Russia from at least the mid-19th century. In any case, Kraevich (1864), (1867), (1874) included an appropriate simple example in these first three editions of his collection of exercises for school students. Both his model and the other one criticized by Markov (below), as well as any other scheme for insuring children, suffered from one and the same essential defect: they were basically unfavourable to the insured (see Section 1; fair insurance is only possible in textbooks), and they did not really insure children. Here is a relevant passage whose author mentions, among other types of insurance, the insurance of children against death (Nikolsky (1895, p. 243)):

Cases that, under the guise of insuring life, conceal deals in paying out some funds upon the occurrence of a stipulated event not inflicting (pecuniary) loss on the insured – deals which do not restore actual damage – should not be attributed to insurance. They abuse the idea of insurance.

Markov’s criticism was justified; regrettably, however, he did not take the occasion to explain to his readers that there existed other forms of life (and property) insurance which were advantageous for the insured.

Elsewhere Markov (1908, p. 97) when referring to his letters, contrasted various forms of insurance:

There exist also (!) such insurance operations which do not protect against any risks, and in all cases inflict some greater or lesser damage on the insured. Such operations may be justified (…) only by a rather doubtful consideration that they compel people to save money.

Note, however, that parents (when juvenile insurance is concerned) become directly interested financially in the survival of their insured children.

5. Markov’s letters


In order to ascertain the benefits of insuring profits and capitals through the state savings offices (…) it is necessary to consider the tariffs. Judging by the number of these (5–10), the
insurance of juveniles plays a large part in the new business direction of the savings offices. Who will benefit from this insurance, excepting ‘those engaged in this operation’? (Markov bears in mind the employees of the offices, see Letter 2.) To answer this question it is necessary to dwell on the insurance tariffs from which we extract two lucid examples.

1. According to Tariff 6, a downpayment of 1200 rubles is necessary for a six-year-old child to draw 2000 rubles after reaching the age of twenty; and, should the child die prematurely, only $1200 - 60 = 1140$ rubles are returned (5% is retained to cover expenses). On the other hand, if the same sum of 1200 rubles is kept at a bank with an interest rate of 4% (this is the rate underlying the tariffs) for each full hundred rubles (interest was paid on the sum rounded down to the nearest hundred) then, consecutively,

$$1200 + 4\% = 1248 \text{ at seven years, (…)}$$
$$1964 + 76 = 2040 \text{ at twenty years.}$$

(Markov had written down all the intermediate results (here omitted).) The same will be true in two other cases below. A rough check of his final figure is provided by calculating

$$1200 \times 1.04^{14} = 2078.$$ 

My calculations show that this insurance is in all cases disadvantageous to the family. If the child survives until age 20, the loss will be expressed by a small sum of 40 rubles; otherwise, it can amount to several hundred rubles since the family loses the interest on the downpayment.

2. According to Tariff 8, a yearly grant of 600 rubles during five consecutive years will be paid out to a child of age six after his reaching age 18 for a downpayment of 1789 rubles; and, should the child die before that age,

$$1789 - 89 = 1700$$ 

rubles are returned. Then

$$1789 + 68 = 1857 \text{ at seven years, (…)}$$
$$2729 + 108 = 2837 \text{ at eighteen years.}$$

So, when paying out the 600 rubles for five years, we obtain, consecutively,

$$2837 - 600 = 2237, (…)$$
$$637 - 600 + 24 = 61.$$ 

It is seen that this operation also inflicts a loss on the family. In the favourable case this loss is expressed by the small sum of 61 rubles; otherwise, it can amount to 1000 rubles.

As indicated above, I have chosen lucid examples, but similar results are obtained in the other cases as well, with the only difference being that, for the most favourable instances, the small loss can become a small profit. However, the possibility of large losses to the family due to the premature death of its child persists.

The explanations provided by the Directorate of the savings offices (Anonymous (1906)) do not explain anything; on the contrary, they obscure the essence of the problem that I have raised. (Markov refers to his Letter 1: the Directorate maintained that the insurance of juveniles ensured but little profit: it ‘comes close (…) to simple saving’. Then, it stated that private insurance societies offer even worse conditions for the insured; the psychological aspect of being protected from chance by insurance is important; if, after some time, the savings offices show a profit higher than 4%, then the surplus will be given over to those insured.) They are composed as though the whole matter consists in the high cost of insurance through savings offices (…). Dwelling only on this small loss, the Directorate maintains that it is of no consequence owing to the security of savings through the insurance, and is compensated by profit sharing. And, assuming an interest rate of 5% rather than 4%, the Directorate promises the insured a payout of 2180–2280 instead of the (promised) 2000.

Thus, the Directorate completely overlooks those cases in which the insured child dies prematurely, and the family’s loss due to the insurance is expressed already in hundreds rather than tens of rubles. Only by forgetting these instances is it possible to state that the savings are here secured. Meanwhile, in my first Note, I had paid attention to these cases; and, for determining the loss incurred by the insurance to the family, I had pointed out, in addition to the figure 2040 on which the Directorate concentrates, a number of other ones. The Directorate apparently chose only the most favourable case; and it vainly tries to prove that in this instance the family’s loss can be replaced by some profit. Indeed, I had mentioned this possibility in my Note: it suffices to change the age of the insured and the duration of the contract in such a way that the probability of losing the stipulated insurance increases.

As to the method by which the Directorate attempts to replace the loss by a profit, it cannot be called proper; this is not only because, instead of providing a detailed calculation, it only indicates an indefinite magnitude between 2180 and 2280, but mainly because it admits that its estimation was based on a change in interest rate. The Directorate obviously forgot that after 14 years and assuming a 5% yearly interest rate, a capital of 1200 rubles, when saved without any insurance being involved, fetches not 2040, but 1200 \times 1.05^{14} = 2374 rubles. (More correctly, 2376. It is obvious that the restriction concerning the interest (see example 1 of Letter 1) did not apply to the savings offices themselves.)

Nevertheless, I am quite prepared to agree with the Directorate that my calculations were based on too low a rate of interest, witness for example the latest pleasing loan. (Markov possibly referred to the ‘Short-Term Treasury Bonds’ issued on 9 December 1905 and yielding a 5.5% rate of interest (Anonymous (1907, p. 67)).) But an increase in the rate increases the family’s loss incurred by the insurance. The Directorate’s statement about an exaggeration in my reckoning is therefore absolutely wrong. Thus, my indication that some insurance procedures offered by the savings offices always lead to losses remains unshaken. Neither can it be shaken until the mortality table taken as the basis for computing the insurance tariffs remains unaltered and the expenses (5%) of carrying out the insurance are not lowered.

Indicating profit sharing, the Directorate says that five years after the insurance operation begins, profit will be shared among the insured; but it forgets to mention that a considerable part (25%) of the profit will go to the employees of the savings offices.

Defending its future operations of insurance for juveniles, the Directorate refers to (private) insurance societies where such operations are carried out according to higher tariffs, but, regrettably, it does not corroborate this statement by comparative excerpts. My remarks
undoubtedly concern these societies as well, but it is also obvious that insurance societies aim at getting rich, and this distinct goal can serve as a warning to those seeking insurance. On the other hand, the fact that some operations are being carried out, is no proof that they should be carried out. For example, a lot of people gather to play roulette in Monaco – so should not we therefore arrange that game, or something similar, at the savings offices? For anyone who read my first Note it should be clear that all the conclusions contained there only concern the insurance of juveniles. The insurance tariffs as carried out by the savings offices cover, however, not one single form of insurance, as might be understood from the words of the Directorate, but several forms, so that, according to the number of the tariffs involved, the insurance of juveniles occupies a rather considerable place among the new operations of the savings offices. And I have provided examples concerning two different tariffs.

I have not touched on other kinds of insurance so that the Directorate apparently defends them in vain; and the less so since its arguments reduce to a statement that for 70 rubles it is possible, given some conditions, to draw 1000 rubles. Is the Directorate really so naïve as to attach serious meaning to this proposition? Having 70 rubles and playing the game of roulette, it is possible to win even more than 1000 rubles.

References


