THE COLLINS CASE: A ‘BEADS IN URNS’ MODEL

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Abstract

The Collins case is an example of the gross misapplication of probability in a legal or forensic context. As such, it has been discussed by a variety of authors who lay bare the fallaciousness of the pseudo-probabilistic reasoning which was applied in the case. The reasoning which ought to be applied in such cases has, however, not been supplied in a clear and explicit manner. In this note the Collins case is described, giving references to some of the discussions of it which have been published. It is then shown how the ‘beads in urns’ paradigm leads us quite simply to the appropriate probability model. The calculations necessary to produce the answer to the relevant probability question turn out to be completely elementary. The same analysis applies directly to the often controversial issue of assessing DNA evidence.

Keywords: Probability model; conditional probability; law of total probability; binomial distribution; forensic probability, DNA

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1. Introduction

A problem involving simple ideas, which are often ill-explained and appear to remain widely misunderstood, arises with respect to the probability of a ‘match’ in the forensic context. A striking example of this problem is provided by the ‘Collins case’ alluded to in the title of this paper. Exactly the same issues are involved in the calculation of probabilities associated with DNA evidence. The issues are thus topical and important; they also appear to lead to a great deal of confusion, the consequences of which are potentially very serious.

In what follows the Collins case will be described (Section 2) and references provided to some of the analyses of this case which have appeared in the literature. We will then (Section 3) use the ‘beads in urns’ paradigm to derive the model which answers the appropriate probability question. We apply this model to the Collins case and to another simple example. Finally (Section 4), we shall state a few general conclusions that may be drawn.

2. Background to the Collins case

The Collins case has been described in a variety of sources. These include the wonderful little book Innumeracy [6, p. 39 ff.] (although the author does not refer to the case by that name), and the books [1, p. 215], [5, p. 219] and [4]. A related discussion (focused on the ideas of DNA matching) is to be found in the review [8].

The Collins case, a wrongful conviction for robbery, transpired in California in 1968. The conviction was based in large part on completely spurious probabilistic arguments. Happily, the spuriousness was eventually recognised and the conviction was overturned by the California
Supreme Court. The basic elements of the case are as follows. Eyewitnesses to a robbery testified that it was committed by a couple consisting of a black man with a beard and moustache and a white woman with blonde hair tied in a pony-tail. The couple were seen driving in a partly yellow car. A couple which matched this description were arrested, but denied the offence and could not otherwise be positively identified.

In court, a mathematics lecturer testified for the prosecution that the six main characteristics had probabilities as follows:

- Woman with blonde hair: $\frac{1}{3}$
- Man with moustache: $\frac{1}{4}$
- Black man with beard: $\frac{1}{10}$
- Woman with ponytail: $\frac{1}{10}$
- Partly yellow car: $\frac{1}{10}$
- Interracial couple in car: $\frac{1}{1000}$

This witness then testified that the ‘product rule’ of probability theory could be used to multiply these probabilities together to obtain a probability of $\frac{1}{12,000,000}$ that a couple chosen at random would have all of these characteristics. The jury was asked to infer that there was therefore only a chance of 1 in 12 million that the defendants were innocent.

Since we simply wish to use the Collins case to illustrate the correct calculation of a certain type of probability, we shall ignore the fact that no substantiation whatsoever was offered for the foregoing list of probabilities. Likewise, we shall overlook the obvious fact that even if these probabilities were correct, multiplying them to obtain the joint probability would be completely unjustified in view of their overwhelming lack of independence. We shall suppose, as do other authors who have discussed this case, that the $\frac{1}{12,000,000}$ probability is actually ‘correct’.

The mistake which we wish to concentrate upon is that of interpreting this $\frac{1}{12,000,000}$ probability as the probability of the defendants’ innocence. This is clearly wrong, and other authors have eloquently explained why it is wrong. In particular, Paulos [6] notes that this mistake amounts to confusing the probability of the evidence, given the guilt of the accused, with the probability of guilt given the evidence. In a later book [7, pp. 72, 73], Paulos discusses this sort of error in the context of calculating probabilities relating to DNA matching, and points out its relation to the logical fallacy of confusing the proposition ‘$p$ implies $q$’ with its converse ‘$q$ implies $p$’.

3. The probability of innocence

In order to reduce the occurrence of the sort of misuse of probability which is exemplified by the Collins case, it is important to show how probability theory may be used correctly in such instances. In turn, this requires that we arrive at a clear understanding of what question we really should be asking and answering. Quite simply, this question is: what is the probability that the suspect is innocent (or guilty), given the evidence? This is not the question answered by any of the analyses referred to in Section 2.

In their discussions, Paulos [6] and Finkelstein and Levin [4] essentially recapitulate the argument given by the defence attorney in the successful appeal of the case. The defence lawyer asked and answered the question ‘what is the probability that there are two such couples, given that there is at least one?’ Chatfield’s argument [1, p. 221] is somewhat different. He effectively asks and answers the question ‘what is the probability that the accused couple is the guilty couple, given that the number of couples matching the description is exactly equal to the expected number of such couples?’

The discussion of the case by Freedman et al. [5] is peculiar. They make no real effort to analyse why the prosecution’s probabilistic reasoning is flawed. Instead, they imply that
the concept of probability does not apply to ‘a unique event that either happened—or didn’t happen’. Embarrassment about having made a similar assertion led David Blackwell to embrace the concepts of Bayesian inference (see [2, p. 43]).

In formulating and answering the appropriate question, many cobwebs are cleared away if we express the problem in terms of a ‘beads in urns’ model (so much the bane of reluctant students in introductory statistics courses). We suppose that there is a (large) urn containing a number of white, and possibly some red, beads. A red bead (the couple seen by the eyewitnesses or the person who left his DNA at the scene of the crime) is observed and then is put back (i.e. escapes) into the urn.

Later, a red bead is drawn from the urn. What is the probability that it is the guilty red bead, i.e. the same red bead that was first seen? Our purpose here is to calculate the probability based only upon the ‘matching’ (red beads), i.e. ignoring any other evidence in the case. In the formalism of the ‘beads in urns’ model, this translates to the assumption that the second bead is drawn at random from the red beads in the urn.

At this stage we make use of conditional probability and the law of total probability (see e.g. [3, p. 81]). If we condition on there being \( n \) red beads in the urn, then the probability that the second bead is the same as the first is trivial to calculate: it is \( 1/n \). But we do not know the number of red beads in the urn; it is a random variable. Let us denote this random variable by \( X \). We need to calculate the probability that the second bead is the same as the first, given that there is at least one bead, i.e. that \( X \geq 1 \).

Let \( G \) be the event that the accused is guilty; by the law of total probability

\[
P(G \mid X \geq 1) = \sum_{n=1}^{N} P(G \mid X = n) P(X = n \mid X \geq 1) = \sum_{n=1}^{N} P(G \mid X = n) \frac{P(X = n)}{1 - P(X = 0)},
\]

where \( N \) is the total number of beads in the urn.

We have already noted that \( P(G \mid X = n) = 1/n \). The assertion that ‘the probability that a randomly chosen couple has the specified characteristics’ (or ‘the probability of a DNA match’) is \( p \), is formalised by saying that the beads in the urn form a random sample of size \( N \) drawn from an infinite population with proportion \( p \) of red beads. The assertion that ‘the probability of a match is \( p \)” tacitly assumes that \( p \) is constant, i.e. that the draws are independent. (A serious assessment of the validity of the independence assumption requires thinking about the mechanism by which the ‘beads’ are actually produced. In this case any dependence we could imagine is likely to be very weak.) Therefore, we may reasonably assume that \( X \) has a binomial distribution with parameters \( N \) and \( p \).

The quantities \( P(X = n) \) can be then be expressed in a straightforward manner, giving

\[
P(G \mid X \geq 1) = \sum_{n=1}^{N} \frac{1}{n} \binom{N}{n} p^n (1 - p)^{N-n} \frac{P(X = n)}{1 - P(X = 0)},
\]

where \( \lambda = Np \).

In the real-life applications of such a model, \( p \) is almost always very small (like \( 1/12000000 \)) and \( N \) is usually very large (like \( 2000000 \)). The circumstances are therefore ideal for using the Poisson approximation to the binomial distribution (see e.g. [3, p. 135]). This gives

\[
P(G \mid X \geq 1) \approx \sum_{n=1}^{N} \frac{e^{-\lambda} \lambda^n / n!}{n} \frac{1}{1 - e^{-\lambda}}, \quad \text{where } \lambda = Np.
\]
This expression cannot be simplified further. For relatively small values of the variables, it is approximately linear in each of $N$ and $p$ (with the other variable held fixed). The linear approximation is $P(G \mid X \geq 1) \approx 1 - 0.25Np$. Be that as it may, (1) is easily evaluated using a computer program such as can be written in S-PLUS® (see for instance [9]) which is given in the appendix. Note that the sum being computed must be truncated. However, a bound on the resulting truncation error is easily calculated. It turns out to be $2.44 \times 10^{-9}$ for the values of $N$ and $p$ proposed in the Collins case, and for similar values will be likewise vanishingly small.

Using (1), the prosecution’s value of $p = 1/12\,000\,000$ and the defence attorney’s value of $N = 2\,000\,000$, we find that in the Collins case the probability of guilt is 0.9587, or that the probability of innocence is 0.0413, which is about half of the 0.08 probability that the defence attorney calculated, but still large enough to acquit. (The linear approximation gives a probability of innocence equal to 0.0417.) Using Chatfield’s value of $N = 24\,000\,000$, the probability of innocence is calculated to be 0.4234, which is close to 0.5, the probability which Chatfield proposes. With such a large $N$ we are not making a particularly egregious error by conditioning on $X = E(X)$. Coincidentally, Chatfield’s probability is exactly equal to the linear approximation.

In Poole’s review [8] of Bennett’s book, an example is given pertaining to DNA matching. Poole uses values $p = 10^{-8}$ and $N = 5 \times 10^9$. From these we calculate a probability of innocence equal to 0.9795, which is almost exactly equal to the value of 0.98 that Poole obtained using reasoning similar to Chatfield’s.

4. Discussion and conclusions

The calculations necessary for finding the probability of ‘innocence given a match’ are in fact trivial. The only difficulty appears to lie in formulating a clear idea of just what event it is whose probability should be calculated. The formulation is facilitated if distracting details are reduced to a minimum. This is why reducing the problem to one of the ‘beads in urns’ sort is so useful.

Moreover, the ‘beads in urns’ model makes clear the crucial role that the size $N$ of the suspect population plays in the calculations. This role is as important as that of $p$, the probability that a random individual has the specified characteristic (e.g. DNA pattern). The dependence of the probability of innocence upon $N$ and $p$ is illustrated in Figure 1. Note the linearity in each

![Figure 1](image-url)
variable (for relatively small values) with the other variable held fixed. The crucial dependence upon \( N \) appears to be largely neglected in discussions of the probability of guilt or innocence based upon DNA or similar evidence. A rational assessment of the value of \( N \) should be mandatory in any trial involving such evidence.

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Appendix A. S-PLUS code

colfun <- function(N,p) {
  # Function to calculate
  # \[ \frac{1}{N} \sum_{n=1}^{\infty} \frac{n}{P(n)} \]
  # where \( P(n) \) are Binomial(N,p) probabilities, using the
  # Poisson approximation to the binomial distribution.

  lambda <- N*p
  summ <- 0
  n <- 1
  xincr <- 1
  epsilon <- .Machine$double.eps
  repeat {
    xincr <- lambda*xincr/n
    summ <- summ + xincr/n
    if(xincr < epsilon | n==N) break
    n <- n + 1
  }
  summ*exp(-lambda)/(1-exp(-lambda))
}

References